Midterm test
Radiation Physics 2018-2019

Friday 5-10-2018: 9:00-11:00

NB5114.004

Read these instructions carefully before making the midterm test!

- **Write your name and student number on *every* sheet.**
- ***Make sure to write readable for other people than yourself. Points will NOT be given for answers in illegible writing.***
- ***Language*; your answers have to be in English.**
- **Use a *separate* sheet for each problem.**
- **Use of a (graphing) calculator is allowed.**
- **This midterm test consists of 3 problems.**
- **The weight of the problems is Problem 1 (P1=30 pts); Problem 2 (P2=30 pts); Problem 3 (P3=30 pts). Weights of the various subproblems are indicated at the beginning of each problem.**
- **The grade of the midterm test is calculated as (P1+P2+P3+10)/10.**
- **For all problems you have to write down your arguments and the intermediate steps in your calculation, *else the answer will be considered as incomplete and points will be deducted.***

PROBLEM 1 *Score: a+b+c+d+e=6+6+6+6+6=30*

Polonium $^{210}_{84}\text{Po}$ is an isotope that alpha decays (100%, half-life: 138 days) to stable $^{206}_{82}\text{Pb}$. The major radiations are given as:

α: 4604 keV (0.00124%); 5407 keV (99.99876%)
γ: 803 keV (0.00123%)

a) Calculate the Q-value for the alpha decay.
b) Draw the decay scheme of $^{210}_{84}\text{Po}$.
c) Explain the process of internal conversion. Calculate the internal conversion coefficient for the gamma transition in the decay of $^{210}_{84}\text{Po}$.
d) Estimate the kinetic energy T_{Pb} of the recoiling lead nucleus after the decay that occurs most frequently. Assume that the $^{210}_{84}\text{Po}$ atom was at rest before the decay.
e) Calculate the power (in Watt) generated by 1 gram of $^{210}_{84}\text{Po}$.

Mass differences Δ (in MeV) of some isotopes

Isotope	Mass difference
$^{210}_{84}\text{Po}$	-15.9531
$^{206}_{82}\text{Pb}$	-23.7854
$^{4}_{2}\text{He}$	2.4249

PROBLEM 2 *Score: a+b+c+d+e =6+7+3+7+7=30*

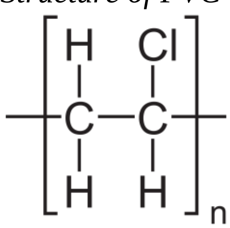
Consider the stopping of protons in polyvinylchloride (PVC). The linear stopping power S (in MeV cm^{-1}) of a charged particle is given by:

$$S = -\frac{dE}{dx} = \frac{5.08 \cdot 10^{-25} z^2 n}{\beta^2}[F(\beta) - \ln I]$$

with $F(\beta)$ a function of which values are given in the table below and z the charge of the particle, n the electron density of the target material (in cm^{-3}) and I the mean excitation energy of the target material (in eV).

a) The electron density of PVC is $4.25 \cdot 10^{23}\,\mathrm{cm}^{-3}$. Calculate the mass density of PVC.
b) Calculate the mean excitation energy of PVC.
c) What are the units of the factor $5.08 \cdot 10^{-25}$?
d) Calculate the stopping power of a 100 MeV proton in PVC.
e) Calculate the stopping power of an 80 MeV alpha particle in PVC. You may assume that the neutron and proton mass are equal.

Structure of PVC



Element	Z	A	I (eV)
H	1	1	19.2
C	6	12	78.0
Cl	17	35.5	174

β^2	$F(\beta)$ Eq. (5.34)
0.008476	9.066
0.01267	9.469
0.01685	9.753
0.02099	9.972
0.04133	10.65
0.08014	11.32
0.1166	11.70
0.1510	11.96
0.1834	12.16
0.3205	12.77
0.5086	13.36
0.6281	13.73
0.7088	14.02
0.7658	14.26

PROBLEM 3 *Score: a+b+c =10+10+10=30*

In case a radionuclide P (decay constant λ_{P}) decays to the radionuclide D (decay constant λ_{D}) which itself decays to E, the time evolution of the number of radionuclides of type D is given by:

$$N_{\mathrm{D}}(t) = \frac{\lambda_{\mathrm{P}} N_P(0)}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\left(e^{-\lambda_{\mathrm{P}} t} - e^{-\lambda_{\mathrm{D}} t}\right) + N_{\mathrm{D}}(0) e^{-\lambda_{\mathrm{D}} t}$$

when initially ($t = 0$) there are $N_{\mathrm{P}}(0)$ radionuclides of type P and $N_{\mathrm{D}}(0)$ of type D.

a) Show that if the half-life of P is many orders of magnitude larger than the half-life of D the activity of D, $A_{\mathrm{D}}(t)$, can be expressed as:

$$A_{\mathrm{D}}(t) = A_{\mathrm{P}}(t)\left(1 - e^{-\lambda_{\mathrm{D}} t}\right) + A_{\mathrm{D}}(0) e^{-\lambda_{\mathrm{D}} t}$$

Consider the following decay chain, in which the half-lives are indicated above the arrows,

$${}^{210}_{82}\mathrm{Pb} \xrightarrow{22y} {}^{210}_{83}\mathrm{Bi} \xrightarrow{5d} {}^{210}_{84}\mathrm{Po} \rightarrow \cdots$$

Consider a sample that contains initially ($t = 0$) 60 MBq of ${}^{210}_{82}\mathrm{Pb}$ and 30 MBq of ${}^{210}_{83}\mathrm{Bi}$.

b) Calculate the specific activity of ${}^{210}_{82}\mathrm{Pb}$ at $t = 0$.
c) Estimate the time at which the activity of ${}^{210}_{83}\mathrm{Bi}$ is 99% of the activity of ${}^{210}_{82}\mathrm{Pb}$. What is this time if the initial activity of ${}^{210}_{83}\mathrm{Bi}$ would have been zero?

Solutions

PROBLEM 1

Solutions

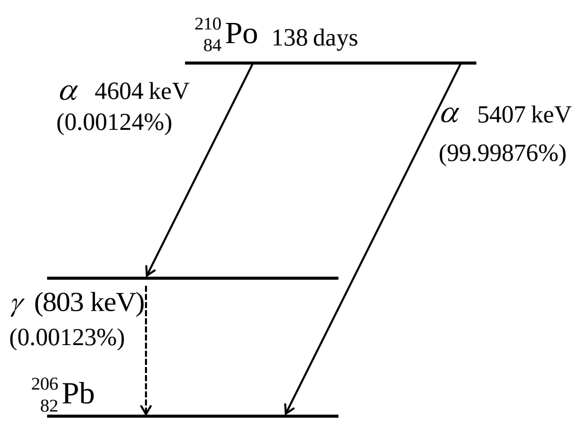
a)

The Q-value for alpha decay is given by $Q_\alpha = \Delta_\mathrm{P} - \Delta_\mathrm{D} - \Delta_\mathrm{He}$. The mass differences are given in the table in the problem.

$$Q_\alpha = \Delta({}^{210}_{84}\mathrm{Po}) - \Delta({}^{206}_{82}\mathrm{Pb}) - \Delta({}^{4}_{2}\mathrm{He}) = -15.9531 - (-23.7854) - 2.4249$$
$$= 5.4074\ \mathrm{MeV}$$

b)

From the Q-value we conclude that the most energetic alpha particle is emitted in a decay directly to the ground state of ${}^{206}_{82}\mathrm{Pb}$. The less energetic alpha particle is emitted in a decay to an excited state of ${}^{206}_{82}\mathrm{Pb}$. This excited state decays mainly by gamma emission. The energy of the gamma is $5407 - 4604 = 803$ keV. In drawing the scheme remember to draw arrows of processes that increase Z to the right and that decrease Z to the left.



c)

Internal conversion is the process in which the energy of an excited nuclear state is transferred to an atomic electron, ejecting it from the atom. The kinetic energy of the ejected electron is the energy of the excited nuclear state minus the binding energy of the electron.

The internal conversion coefficient α_{IC} is defined as the ratio of the number of conversion electrons to the number of gammas.

In this case we have: $\alpha_{IC} = \frac{0.00001}{0.00123} = 8.1 \cdot 10^{-3}$

d)

The Q-value energy is shared after the decay by the recoil nucleus (mass M, velocity V) and the alpha particle (mass m, velocity v). The Po atom was at rest before the decay, thus the total momentum was zero. This means that after the decay we have (conservation of momentum),

$$mv = MV$$

Conservation of energy gives,

$$Q = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Combining these equations leads to:

$$T_{Pb} = \frac{1}{2}MV^2 = \frac{mQ}{m+M} = \frac{4}{206+4}Q = 0.103\text{MeV}$$

e)

In each decay about 5.4 MeV is released. The number of atoms in 1 gram of ^{210}Po is,

$$N_{\text{Po}} = \frac{1}{210} \times 6 \cdot 10^{23} = 2.86 \cdot 10^{21}$$

The number of decaying ^{210}Po atoms per second is:

$$\lambda_{\text{Po}} N_{\text{Po}} = \frac{\ln 2\, N_{\text{Po}}}{T_{\frac{1}{2}}(\text{Po})} = \frac{0.693 \times 2.86 \cdot 10^{21}}{138 * 24 * 3600} = 1.66 \cdot 10^{14}\ \text{s}^{-1}$$

Total energy production per second (P) is:

$$P = 1.66 \cdot 10^{14} \times 5.4 \cdot 10^{6} = 8.96 \cdot 10^{20}\ \text{eV s}^{-1} = 8.96 \cdot 10^{20} \times 1.6 \cdot 10^{-19}$$
$$= 143\ \text{J s}^{-1} = 143\ \text{W}$$

PROBLEM 2

Solution

a)

Each unit of PVC has 2x6 (Carbon) plus 3x1 (Hydrogen) plus 1x17 (Chlorine) is 32 electrons. The molar mass of such a unit is 2x12 (Carbon) plus 3x1(Hydrogen) plus 1x35.5 (Chlorine) is 62.5 gram per mole. The electron density n of PVC is given as $4.25 \cdot 10^{23}$ cm^{-3}. The mass density ρ (in g cm^{-3}) can be found from:

$$n = \frac{32}{62.5} \times \rho \times N_a = \frac{32}{62.5} \times \rho \times 6 \cdot 10^{23} = 4.25 \cdot 10^{23} \text{ cm}^{-3} \Rightarrow$$

$$\rho = \frac{4.25 \cdot 10^{23}}{\frac{32}{62.5} \times 6 \cdot 10^{23}} = 1.38 \text{ g cm}^{-3}$$

b)

Use Bragg's rule

$$\ln I_{PE} = \frac{\sum_i w_i \frac{Z_i}{A_i} \ln I_i}{\sum_i w_i \frac{Z_i}{A_i}}$$

With $w_H = \frac{3}{62.5}$ and $w_C = \frac{24}{62.5}$ and $w_{Cl} = \frac{35.5}{62.5}$ this becomes:

$$\ln I_{PVC} = \frac{\frac{3}{62.5}\frac{1}{1}\ln 19.2 + \frac{24}{62.5}\frac{6}{12}\ln 78 + \frac{35.5}{62.5}\frac{17}{35.5}\ln 174}{\frac{3}{62.5}\frac{1}{1} + \frac{24}{62.5}\frac{6}{12} + \frac{35.5}{62.5}\frac{17}{35.5}}$$

$$= \frac{0.048 \times 2.95 + 0.192 \times 4.36 + 0.272 \times 5.16}{0.048 + 0.192 + 0.272} = \frac{2.38}{0.512} = 4.65$$

$$I_{PVC} = 104 \text{ eV}$$

c) The units are Mev cm^2.

d)

For a 100 MeV proton we have

$$T = (\gamma - 1)m_p c^2 \Rightarrow 100 = (\gamma - 1)938 \Rightarrow \gamma = 1 + \frac{100}{938} = 1.107$$

and

$$\beta^2 = 1 - \frac{1}{\gamma^2} = 0.183$$

and from the table we have $F(\beta) = 12.16$.

$$S = \frac{5.08 \cdot 10^{-25} \times 1 \times 4.25 \cdot 10^{23}\ \mathrm{cm}^{-3}}{0.183}[12.16 - 4.65] = 8.9\ \mathrm{MeV\ cm}^{-1}$$

e)
For a 80 MeV alpha particle we have,

$$T = (\gamma - 1)m_\alpha c^2 \Rightarrow 80 = (\gamma - 1) \times 4 \times 938 \Rightarrow \gamma = 1 + \frac{80}{4 \times 938} = 1 + \frac{20}{938} = 1.021$$

$$\beta^2 = 1 - \frac{1}{\gamma^2} = 0.041$$

and from the table we have $F(\beta) = 10.65$.

So with $z^2 = 4$ instead of $z^2 = 1$ in the equation of the stopping power we find:

$$S = \frac{5.08 \cdot 10^{-25} \times 4 \times 4.25 \cdot 10^{23}\ \mathrm{cm}^{-3}}{0.041}[10.65 - 4.65] = 126\ \mathrm{MeV\ cm}^{-1}$$

PROBLEM 3

a)

Start with

$$N_{\mathrm{D}}(t)=\frac{\lambda_{\mathrm{P}}N_P(0)}{\lambda_{\mathrm{D}}-\lambda_{\mathrm{P}}}\left(e^{-\lambda_{\mathrm{P}}t}-e^{-\lambda_{\mathrm{D}}t}\right)+N_{\mathrm{D}}(0)e^{-\lambda_{\mathrm{D}}t}\Rightarrow$$

$$\lambda_{\mathrm{D}}N_D(t)=\frac{\lambda_{\mathrm{D}}\lambda_{\mathrm{P}}N_P(0)}{\lambda_{\mathrm{D}}-\lambda_{\mathrm{P}}}\left(e^{-\lambda_{\mathrm{P}}t}-e^{-\lambda_{\mathrm{D}}t}\right)+\lambda_{\mathrm{D}}N_D(0)e^{-\lambda_{\mathrm{D}}t}\Rightarrow$$

$$A_{\mathrm{D}}(t)=\frac{\lambda_{\mathrm{D}}\lambda_{\mathrm{P}}N_P(0)}{\lambda_{\mathrm{D}}-\lambda_{\mathrm{P}}}\left(e^{-\lambda_{\mathrm{P}}t}-e^{-\lambda_{\mathrm{D}}t}\right)+A_{\mathrm{D}}(0)e^{-\lambda_{\mathrm{D}}t}$$

The half-life of P is many orders of magnitude larger than the half-life of D thus

$$T_{\frac{1}{2}}(P)\gg T_{\frac{1}{2}}(D)\Rightarrow\lambda_{\mathrm{P}}\ll\lambda_{\mathrm{D}}$$

Consequently, $\lambda_{\mathrm{D}}-\lambda_{\mathrm{P}}\approx\lambda_{\mathrm{D}}$ leading to,

$$A_{\mathrm{D}}(t)=\frac{\lambda_{\mathrm{D}}\lambda_{\mathrm{P}}N_P(0)}{\lambda_{\mathrm{D}}}e^{-\lambda_{\mathrm{P}}t}\left(1-e^{-(\lambda_{\mathrm{D}}-\lambda_{\mathrm{P}})t}\right)+A_{\mathrm{D}}(0)e^{-\lambda_{\mathrm{D}}t}\Rightarrow$$

$$A_{\mathrm{D}}(t)=\lambda_{\mathrm{P}}N_P(0)e^{-\lambda_{\mathrm{P}}t}\left(1-e^{-\lambda_{\mathrm{D}}t}\right)+A_{\mathrm{D}}(0)e^{-\lambda_{\mathrm{D}}t}=A_{\mathrm{P}}(t)\left(1-e^{-\lambda_{\mathrm{D}}t}\right)+A_{\mathrm{D}}(0)e^{-\lambda_{\mathrm{D}}t}$$

b)

The specific activity is the activity per unit mass of the material.

We have an activity A of 60 MBq of ${}^{210}_{82}\mathrm{Pb}$. The number N of atoms ${}^{210}_{82}\mathrm{Pb}$ is

$$N=\frac{A}{\lambda_{Pb}}$$

The mass (in gram) of these N atoms is

$$\frac{N}{6\cdot10^{23}}M=\frac{A}{6\cdot10^{23}}\frac{M}{\lambda_{Pb}}$$

With M the molar mass of ${}^{210}_{82}\mathrm{Pb}$ which is approximately 210 gram. The specific activity (SA) then becomes:

$$\mathrm{SA}=\frac{A}{\frac{A}{6\cdot10^{23}}\frac{M}{\lambda_{Pb}}}=\frac{6\cdot10^{23}\lambda_{Pb}}{M}=\frac{6\cdot10^{23}}{210}\frac{\ln 2}{22\times365.25\times24\times3600}\Rightarrow$$

$$\mathrm{SA}=2.85\cdot10^{12}\ \mathrm{Bq\ g^{-1}}$$

c)

We use the equation from a)

$$A_{\mathrm{D}}(t)=A_{\mathrm{P}}(t)\left(1-e^{-\lambda_{\mathrm{D}}t}\right)+A_{\mathrm{D}}(0)e^{-\lambda_{\mathrm{D}}t}$$

We have to find the time at which we have:

$$0.99 \times A_{\mathrm{P}}(t) = A_{\mathrm{D}}(t) = A_{\mathrm{P}}(t)\left(1 - e^{-\lambda_{\mathrm{D}} t}\right) + A_{\mathrm{D}}(0) e^{-\lambda_{\mathrm{D}} t}$$

We can neglect the decay of P in this approximation and thus $A_{\mathrm{P}}(t) = A_{\mathrm{P}}(0)$.

Applying this to the ${}^{210}_{82}\mathrm{Pb}$ and ${}^{210}_{83}\mathrm{Bi}$ situation we have:

$$0.99 \times 60 = 60\left(1 - e^{-\lambda_{\mathrm{D}} t}\right) + 30 e^{-\lambda_{\mathrm{D}} t} = 60 - 30 e^{-\lambda_{\mathrm{D}} t} \Rightarrow$$

$$e^{-\lambda_{\mathrm{D}} t} = \frac{0.01 \times 60}{30} = 0.02 \ \Rightarrow t = \frac{-\ln 0.02}{\lambda_{\mathrm{D}}} = \frac{3.91 \times 5}{\ln 2} \approx 28 \text{ days}$$

In the initial activity of ${}^{210}_{83}\mathrm{Bi}$ would have been zero we have:

$$0.99 \times 60 = 60\left(1 - e^{-\lambda_{\mathrm{D}} t}\right) = 60 - 60 e^{-\lambda_{\mathrm{D}} t} \Rightarrow$$

$$e^{-\lambda_{\mathrm{D}} t} = \frac{0.01 \times 60}{60} = 0.01 \ \Rightarrow t = \frac{-\ln 0.01}{\lambda_{\mathrm{D}}} = \frac{4.61 \times 5}{\ln 2} \approx 33 \text{ days}$$